



TONBRIDGE SCHOOL

Scholarship Examination 2008

MATHEMATICS II

**Wednesday 30th April 2008
2.00 p.m.**

Time allowed: 1 hour 30 minutes

Answer as many questions as you can.

All the questions carry equal marks.

All answers must be supported by adequate explanation.

Calculators may be used in any question.

1. The curved surface area S of a cone of base radius r and height h is given by the formula $S = \pi r \sqrt{r^2 + h^2}$.

- (a) If $r = 11$ cm and $h = 3$ cm, find S .
- (b) If $S = 150$ cm² and $r = 5$ cm, find h .
- (c) If $S = 100$ cm² and the height is twice the base radius, find r .

2. (a) By first adding the three equations together, or otherwise, find the values of A , B , C which satisfy the equations below:

$$A - B - C = -7$$

$$B - C - A = 3$$

$$C - A - B = 5$$

(b) Find the values of A , B , C , D which satisfy the equations:

$$A - B - C - D = -6$$

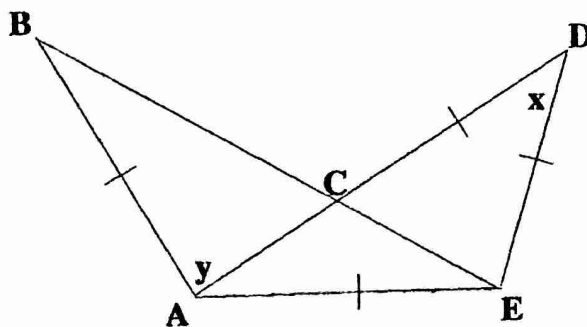
$$B - C - D - A = -2$$

$$C - D - A - B = 4$$

$$D - A - B - C = 8$$

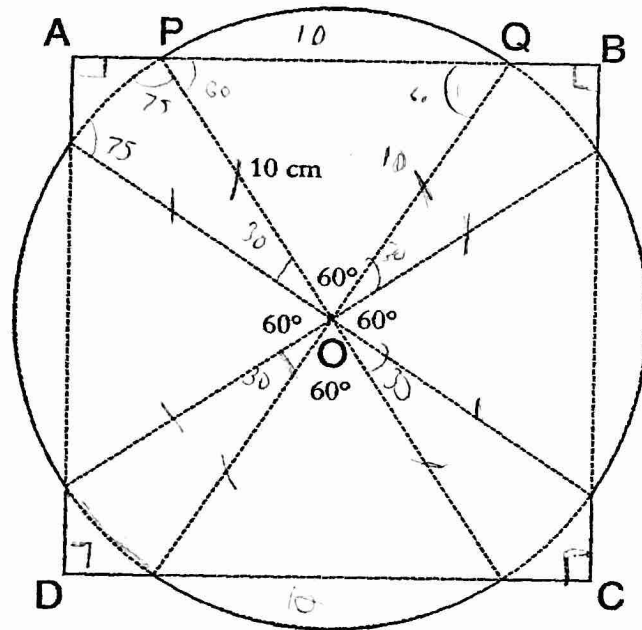
3. In the figure, triangle ABE is isosceles with $AB = AE$ and triangle CDE is isosceles with $DC = DE$. Also $AE = DE$ and BCE and ACD are straight lines. Angle $CDE = x$ and angle $BAC = y$.

- (a) If $x = 20^\circ$, find y .
- (b) Use algebra to find the connection between y and x . You should simplify your answer as far as possible.
- (c) By considering the angles in the figure, explain carefully what happens to the figure if $x = 60^\circ$.



4.

The figure enclosed by the solid lines below consists of a circle of radius 10 cm symmetrically superimposed on a square ABCD. The four sectors of the circle such as OPQ all have angles of 60° as shown (the dotted lines are just there to help you visualise the pieces of the figure).



- (a) Find the side-length of the square ABCD. $\frac{360 - (2 \times 60)}{9}$
 (Hint: What sort of triangle is OPQ?)
- (b) Find the total perimeter of the figure.
- (c) Find the total area of the figure.

5.

(In this question, you are reminded that the curved surface area of a cylinder of radius r and height h is given by the formula $2\pi rh$.)

A solid metal cylinder has base radius 3 cm and height 24 cm.

- (a) Find the total surface area of the cylinder (including both ends).
- (b) The cylinder is split into three identical pieces by two straight cuts parallel to its base: each piece is thus a cylinder with base radius 3 cm and height 8 cm. Find the total surface area of all three pieces.
- (c) An identical cylinder to that in (a) is split into N identical pieces by straight cuts parallel to its base. Find N if the total surface area of all N pieces is double the total surface area of the cylinder in (a).



$$2\pi r^2 + 2\pi r h$$

$$2\pi(9)$$

$$3 \left[2\pi(3)(8) + 2\pi(3^2) \right] =$$

$$60\pi$$

6. This question concerns the graph of $y = (x - \sqrt[3]{x})^2$.

$$\sqrt[3]{x} = x^{1/3}$$

$$\sqrt{x} = x^{1/2}$$

- (a) When $x=3$, show that $y=2.43$ (correct to 2 decimal places).
- (b) Find the values of y corresponding to $x=0, 1, 1.5, 2, 2.5, 3.5$.
- (c) With a scale of 1 unit = 2 cm on each axis, use your values from (a) and (b) to draw the graph of y against x .
- (d) What happens to the graph of $y = (x - \sqrt[3]{x})^2$ for values of x between 0 and 1?
- (e) Draw the line $y=x$ on top of your graph in (c).
- (f) Use your graphs to write down, correct to 1 decimal place, a number (other than zero) with the property that it is equal to the sum of its square root and its cube root.

7. In the table below, the sums in Column A are made by adding together cubes of odd numbers. The total obtained is shown in Column B and a factorisation of this total is shown in Column C.

	A	B	C
Row 1	1^3	1	1×1
Row 2	$1^3 + 3^3$	28	4×7
Row 3	$1^3 + 3^3 + 5^3$	153	9×17
Row 4	$1^3 + 3^3 + 5^3 + 7^3$	496	16×31
Row 5			
Row 6			
Row n			

- (a) Write down the values that are in Columns A, B, C for Rows 5 and 6.
- (b) Study Column C carefully. What is the formula in terms of n for the two numbers that are multiplied together in Column C for Row n ?
- (c) Use your answer to (b) to find:
 (i) The entry in Column B for Row 15.
 (ii) The value of n if Row n has an entry of 780625 in Column B.